Acceleration of the Universe in Presence of Tachyonic field

Surajit Chattopadhyay[1][*], **Ujjal Debnath**[2][†] **and Goutami Chattopadhyay**[2]

[1]*Department of Information Technology, Pailan College of Management
and Technology, Bengal Pailan Park, Kolkata-700 104, India.*

[2]*Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah-711 103, India.*

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In this letter, we have assumed that the Universe is filled in tachyonic field with potential, which gives the acceleration of the Universe. For certain choice of potential, we have found the exact solutions of the field equations. We have shown the decaying nature of potential. From recently developed statefinder parameters, we have investigated the role of tachyonic field in different stages of the evolution of the Universe.

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Recent observations on microwave background radiation have offered good amount of support to cosmological inflation to be an integral part of the standard model of the universe. On the other hand there have been difficulties in obtaining accelerated expansion from fundamental theories such as M/String theory [1]. Much has been written and emphasized about the role of the fundamental dilation field in the context of string cosmology. But, not much emphasized is tachyon component [2]. It has been recently shown by Sen [3, 4] that the decay of an unstable D-brane produces pressure-less gas with finite energy density that resembles classical dust. The cosmological effects of the tachyon rolling down to its ground state have been discussed by Gibbons [5]. Rolling tachyon matter associated with unstable D-branes has an interesting equation of state which smoothly interpolates between −1 and 0. As the Tachyon field rolls down the hill, the universe experiences accelerated expansion and at a particular epoch the scale factor passes through the point of inflection marking the end of inflation [1]. The tachyonic matter might provide an explanation for inflation at the early epochs and could contribute to some new form of cosmological dark matter at late times [6]. Inflation under tachyonic field has also been discussed in ref. [2, 7, 8]. Sami et al [9] have discussed the cosmological prospects of rolling tachyon with exponential potential. In this letter, we have assumed that the Universe is filled in tachyonic field with potential, which gives the acceleration of the Universe.

The action for the homogeneous tachyon condensate of string theory in a gravitational background is given by,

$$S = \int \sqrt{-g} \, d^4x \left[\frac{\mathcal{R}}{16\pi G} + \mathcal{L} \right] \tag{1}$$

where $\mathcal{L}$ is the Lagrangian density given by,

$$\mathcal{L} = -V(\phi)\sqrt{1 + g^{\mu\nu} \, \partial_\mu\phi\partial_\nu\phi} \tag{2}$$

where ϕ is the tachyonic field, $V(\phi)$ is the tachyonic field potential and $\mathcal{R}$ is the Ricci Scalar. The energy-momentum tensor for the tachyonic field is,

$$T_{\mu\nu} = -\frac{2\delta S}{\sqrt{-g} \, \delta g^{\mu\nu}} = -V(\phi)\sqrt{1 + g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}g^{\mu\nu} + V(\phi)\frac{\partial_\mu\phi\partial_\nu\phi}{\sqrt{1+g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}}$$

$$= p \, g_{\mu\nu} + (p + \rho)u_\mu u_\nu \tag{3}$$

where the velocity u_μ is :

$$u_\mu = -\frac{\partial_\mu\phi}{\sqrt{-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}} \tag{4}$$

[*] surajit_2008@yahoo.co.in

[†] ujjaldebnath@yahoo.com , ujjal@iucaa.ernet.in

with $u^\nu u_\nu = -1$.

The energy density ρ and the pressure p of the tachyonic field therefore are,

$$\rho = \frac{V(\phi)}{\sqrt{1 - \dot\phi^2}} \tag{5}$$

and

$$p = -V(\phi)\sqrt{1 - \dot\phi^2} \tag{6}$$

Now the metric of a spatially flat isotropic and homogeneous Universe in FRW model is

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{7}$$

where $a(t)$ is the scale factor.

Here we have assumed that the universe is filled in only tachyonic field, so the Einstein field equations are (choosing $8\pi G = c = 1$) given by

$$3\frac{\dot a^2}{a^2} = \rho \tag{8}$$

and

$$6\frac{\ddot a}{a} = -(\rho + 3p) \tag{9}$$

The energy conservation equation is

$$\dot\rho + 3\frac{\dot a}{a}(\rho + p) = 0 \tag{10}$$

which leads to

$$\frac{\dot V}{V\dot\phi^2} + \frac{\ddot\phi}{\dot\phi}\left(1 - \dot\phi^2\right)^{-1} + 3\frac{\dot a}{a} = 0 \tag{11}$$

Now, in order to solve the equation (11), we take a simple form of $V = \left(1 - \dot\phi^2\right)^{-m}$, $(m > 0)$, so that the solution of ϕ becomes

$$\phi = \frac{2a^{3/2}}{\sqrt{3c}} \; {}_2F_1[\frac{1+2m}{4}, \frac{3+2m}{4}, \frac{5+2m}{4}, -c^{-\frac{2}{2m+1}}a^{\frac{6}{1+2m}}] \tag{12}$$

where c is an integration constant. From figure 1, we see that ϕ increases with a. The potential V of the tachyonic field ϕ can be written as

$$V = \left[1 + \left(\frac{c}{a^3}\right)^{\frac{2}{1+2m}}\right]^m \tag{13}$$

So from equations (12) and (13), we have the relation between ϕ and V as

$$\phi = \frac{1}{\sqrt{3}}\left(V^{\frac{1}{m}} - 1\right)^{\frac{1}{1+1m}} \; {}_2F_1[\frac{1+2m}{4}, \frac{3+2m}{4}, \frac{5+2m}{4}, -(V^{\frac{1}{m}} - 1)^{-1}] \tag{14}$$
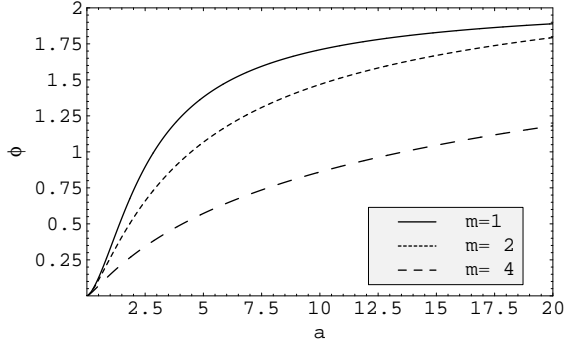
Fig.1

Fig.2
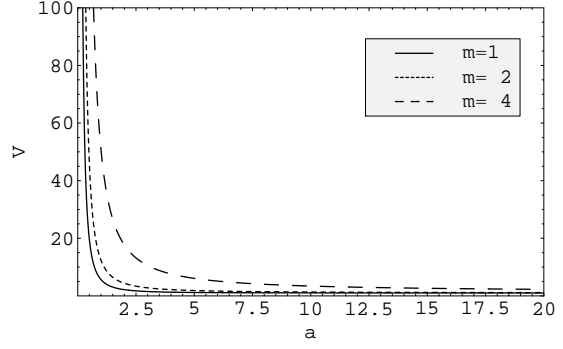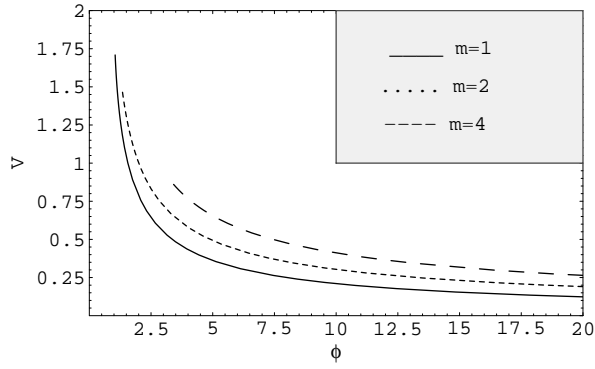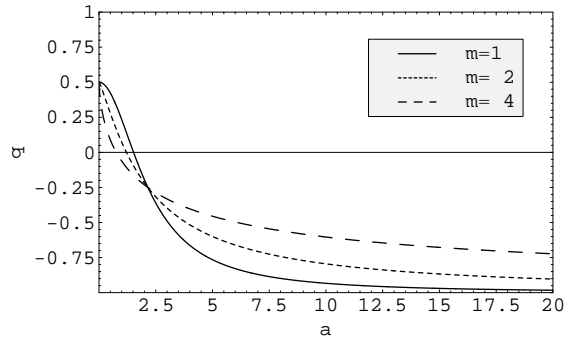

Fig.3

Fig.4

Figs. 1, 2 and 4 show variation of ϕ, V and q against a respectively and fig. 3 shows variation of V against ϕ for different values of m (= 1, 2, 4).

From above expression of V, we see that V decreases from large value to 1 as a increases, which is shown in graphically in figure 2. From figure 3, we see that $V(\phi)$ decreases as ϕ increases i.e., V decreases with evolution of the universe.

Also from equation (8), we have the solution for a as

$$t = \frac{2}{\sqrt{3c}} \, a^{\frac{3}{2}} \, {}_2F_1\Big[\frac{1+2m}{4}, \frac{1+2m}{4}, \frac{5+2m}{4}, -c^{-\frac{2}{2m+1}} a^{\frac{6}{1+2m}}\Big] \tag{15}$$

The deceleration parameter q has in the form

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1}{2}\left[1 - \frac{3}{1 + \left(\frac{c}{a^3}\right)^{\frac{2}{1+2m}}}\right] \tag{16}$$

For accelerating universe, q must be negative i.e., $a > c^{\frac{1}{3}} \, 2^{-\frac{2m+1}{6}}$. From figure 4, we see that q decreases from 0.5 to -1 i.e., the sign flip from positive to negative signature of q in matter dominated era which is caused by tachyonic field.

From equations (5) and (6), we have
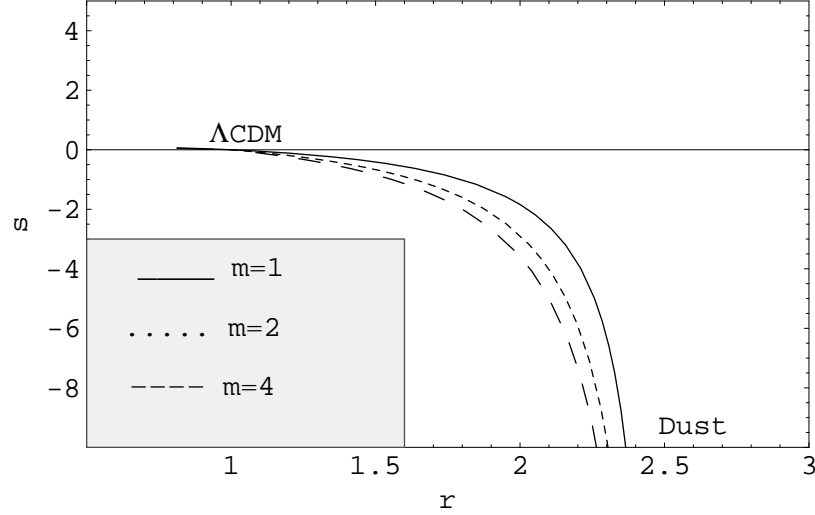
$$w_{eff} = \frac{p}{\rho} = -1 + \dot{\phi}^2 \tag{17}$$

Fig.5

Fig. 5 shows the variation of s against r for different values of m $(=\ 1,\ 2,\ 4)$.

For real values of ρ and p, we must have $\dot{\phi}^2 \leq 1$, which implies $-1 \leq w_{eff} \leq 0$. This interprets that the tachyonic field interpolates between dust and ΛCDM stages. From figure 1, we see that the deceleration parameter q lies between 0.5 and -1 for different values of m i.e., universe starts from dust to ΛCDM model.

The statefinder diagnostic pair $\{r,s\}$, introduced by Sahni et al [10] is constructed from the scale factor $a(t)$ and its derivatives up to the third order as follows:

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} \tag{18}$$

where $H\left(=\frac{\dot{a}}{a}\right)$ is the Hubble parameter. These parameters are dimensionless and allow us to characterize the properties of dark energy in a model independent manner. The parameter r forms the next step in the hierarchy of geometrical cosmological parameters after H and q. From equations (16) and (18) we have the relation between r and s:

$$8(2m-1)(r-1)^2 + 6(14m-5)(r-1)s - 9(1+2m)(2r-5)s^2 = 0 \tag{19}$$

Figure 5 represents the variation of s against r for different values of m $(=\ 1,\ 2,\ 4)$. The negative side of s represents the evolution of the universe starts from dust state $(r = \text{finite},\ s \to -\infty)$ to the ΛCDM $(r = 1,\ s = 0)$ model.

In this letter, we have considered the flat FRW Universe driven by only tachyonic field. We have presented accelerating expansion of our Universe due to tachyonic field. We found exact solutions of tachyonic field and potential by considering specific form of potential. We show that the potential represent the decaying nature as ϕ or a increases. Note that if $V = $ constant, the tachyonic field represents the pure Chaplygin gas. If $V \neq$ constant, the tachyonic field may be treated as variable Chaplygin gas. Graphical representation of q shows that the tachyonic field propagates between dust (early stage) and ΛCDM stages (late stage). $\{r,s\}$ figure shows the evolution of the universe starts from dust and ends at ΛCDM stage. At whole stages of the evolution, r is always ≥ 1 and s becomes negative. Thus from the behaviour of q and $\{r,s\}$, we say that acceleration is possible if the universe is filled in only tachyonic field.

Acknowledgement:

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